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EFFECTS OF MAGNETIC MONOPOLES ON NUCLEAR WAVE FUNCTIONS AND POSSIBLE CATALYSIS OF NUCLEAR BETA DECAY AND SPONTANEOUS FISSION

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ABSTRACT

Mixing effects in nuclear wave functions by the strong magnetic field of a magnetic monopole are estimated. A monopole at a distance of 10 fermis from a deuteron mixes the singlet and triplet spin levels with a strength comparable to the deuteron binding energy. Forbidden nuclear beta decay transitions can be enhanced by mixing nuclear wave functions with other states for which the beta decay transition is less inhibited. Particularly suitable candidates have nearby excited states connected by magnetic dipole transitions to the ground state. Magnetic mixing can also strongly enhance spontaneous fission.

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The magnetic energy of a nuclear spin in the magnetic field produced by a monopole at a distance of several fermis is of the order of nuclear binding energies and level spacings. This field can produce appreciable mixing of nuclear wave functions and may give observable effects. For a rough estimate consider a deuteron in a very strong magnetic field. In the same way that a field splits and mixes the triplet and singlet spin states of positronium, the spin triplet deuteron ground state is split into three energy levels and its central member is mixed with the (unbound) singlet spin state to produce the eigenstates $|n\uparrow p\downarrow\rangle$ and $|n\downarrow p\uparrow\rangle$. The splitting between the two states by the field at a distance of r from a monopole of charge g is

$$\Delta E = g(1.91 + 2.79)(e\hbar/Mc)(1/r^2) \tag{1}$$

where M is the proton mass. If we set $eg/\hbar c = 1$, the splitting is equal to the binding energy of the deuteron, 2 MeV when

$$\hbar c/r = 20 \text{ MeV}. \tag{2a}$$

$$r \simeq 10 \text{ fermis} \tag{2b}$$

This suggests that a monopole might break up a deuteron at a distance of 10 fermis.

However, a slowly-moving heavy monopole does not have the kinetic energy required to break up a deuteron, and the two nucleons either remain bound to the monopole or escape as a normal deuteron. This differs from positronium, where both the triplet and singlet states are metastable and decay by annihilation. The triplet decay is inhibited by selection rules and is third order in α (3γ) while the singlet is second order (2γ). An external magnetic field catalyzes the decay of the triplet state by mixing in a singlet component for which the second order decay is allowed.

The nuclear analog of positronium is a mestastable nuclear state whose decay can be enhanced or catalyzed by the presence of the monopole magnetic field; e.g. beta unstable odd-odd nuclei like Al^{26} whose ground states have the proton spin j_p and the neutron spin j_n coupled to the maximum possible spin $J = j_p + j_n$ and whose beta decay to a J=0 even-even nucleus is highly forbidden because of the large spin change. A strong magnetic field decouples j_p and j_n and mixes in all lower spin couplings down to $J = |j_p - j_n|$. As in triplet positronium, decays from the admixed states have a much lower order of forbiddenness and the decay rate is enhanced.

Spontaneous electromagnetic mixing has been considered [1,2] as a radiative correction to ordinary beta decay and found to be much too small to produce an observable effect. The induced mixing due to a monopole is similar to this radiative mixing, but the transition matrix element is of order unity instead of order α. The transition probability is thus increased by the large factor of α^2.

The enhancement factor in the transition matrix element M_B for the magnetically induced beta decay over the ordinary decay matrix element M_o for various combinations of electromagnetic transitions and one ordinary beta decay is given by standard perturbation theory as

$$\frac{\langle f|M_B|i\rangle}{\langle f|M_o|i\rangle} = \frac{\langle f|V|A\rangle}{\langle f|V|i\rangle} \cdot \frac{\langle A|H_B|i\rangle}{(E_A - E_i)} \tag{3a}$$

$$\frac{\langle f|M_B|i\rangle}{\langle f|M_o|i\rangle} = \frac{\langle B|V|i\rangle}{\langle f|V|i\rangle} \cdot \frac{\langle f|H_B|B\rangle}{(E_B - E_f)} \quad . \tag{3b}$$

$$\frac{\langle f|M_B|i\rangle}{\langle f|M_o|i\rangle} = \frac{\langle B|V|A\rangle}{\langle f|V|i\rangle} \cdot \frac{\langle f|H_B|B\rangle}{(E_B - E_f)} \cdot \frac{\langle A|H_B|i\rangle}{(E_A - E_i)} \tag{3c}$$

where V denotes the transition operator for ordinary beta decay, H_B denotes the electromagnetic transition operator, $|i\rangle$, $|f\rangle$, $|A\rangle$ and $|B\rangle$ denote the initial and final nuclear ground states and the intermediate excited states and E_i, E_f, E_A and E_B denote their energies.

Two types of ratios appear in eqs. (3): the ratio of the beta decay matrix element involving the hopefully more favorable intermediate states to the original beta decay matrix element, and the ratio of the magnetic mixing matrix element to the appropriate energy denominator. In cases of interest, the ratio of beta decay matrix elements is much larger than unity, since intermediate states are chosen to give more favorable transitions. The electromagnetic matrix elements depend upon nuclear wave functions. The most favorable intermediate states are those having strong M1 transitions to the initial or final states, since H_B is essentially an M1 operator.

One estimate of the matrix elements of H_B is obtained from the mixing of the triplet and singlet states of the deuteron, denoted by D_3 and D_1,

$$\langle D_3|H_B|D_1\rangle = g(1.91 + 2.79)(e\hbar/2Mc)(1/r^2) = (1/400)(\hbar c/r)^2 \text{ MeV} \quad (4)$$

for $eg/\hbar c = 1$. For $r = 10$ fermis, this gives 1 MeV, which is of the same order as the energy denominators. This suggests appreciable enhancements in favorable cases for monopoles at distances of the order of 10 fermis from nuclei.

Second-forbidden beta transitions with $\Delta J = 2$ and no parity change can go in two steps as an electromagnetic M1 transition and an allowed Gamow-Teller beta transition. For example, the decay of the $J^P = 2^+$ ground state of Cl^{36} to the 0^+ ground state of S^{36} is second forbidden and has a log ft value of 13.5. Cl^{36} has an excited 1^+ state at 1.2 MeV from which an allowed Gamow-Teller transition could go to the 0^+ ground state of S^{36}. For a crude estimate of this allowed transition we use the log ft of 4.9 for the allowed GT decay of the 3^+ ground state of Al^{28} to the 2^+ state of Si^{28}. The enhancement factor in eq. (3a) due to the beta decay matrix elements is thus four orders of magnitude (eight orders of magnitude in the rate) and considerable enhancement should remain even if the second factor is quite small.

Particularly interesting cases might be odd-odd nuclei with the odd proton and odd neutron in the same L-shell and coupled to a spin of 2 or greater. The beta decay to a 0^+ ground state then involves recoupling the angular momenta of the two nucleons and is forbidden because orbital factors are needed for a change in total angular momentum larger than one. However, the monopole could break the couplings of the proton and neutron spins. In perturbation theory this appears as a cascade of M1 transitions via the other states of the same configuration down to the 1^+ state, from which the beta decay is an allowed GT transition. Thus very long-lived highly forbidden transitions might go much more rapidly via the magnetic transition through several intermediate states. The relevant matrix elements for such transitions can be crudely estimated using shell model wave functions and experimental values of magnetic moments and Gamow-Teller matrix elements within the same configurations.

Consider, for example Al^{26}, which has $J^P = 5^+$ and decays to the excited 2^+ state of Mg^{26} with a lifetime of 7.2×10^5 years and a log ft of 14.2. Al^{26} also has excited states with $J^P = 4^+, 3^+, 2^+$ and 1^+ at excitation energies of 2 MeV, 0.4 MeV, 1.8 MeV and 1.1 MeV respectively. The beta decays from the 1^+ state of Al^{26} to the 0^+ ground state of Mg^{26} and from the 3^+ and 4^+ states of Al^{26} to the 2^+ and 3^+ excited states of Mg^{26} are both allowed GT transitions. Thus a monopole-induced fifth-order transition via four intermediate states or a second or third-order transition via one or two intermediate states might have a much shorter lifetime than the observed decay.

The transition matrix elements for the fifth order, third and second order transitions are

$$\langle 0^+|M_B|5^+\rangle = \langle 0^+|V|1^+\rangle \frac{\langle 1^+|H_B|2^+\rangle}{E(1^+)-E(5^+)} \cdot \frac{\langle 2^+|H_B|3^+\rangle}{E(2^+)-E(5^+)} \frac{\langle 3^+|H_B|4^+\rangle}{E(3^+)-E(5^+)} \cdot \frac{\langle 4^+|H_B|5^+\rangle}{E(4^+)-E(5^+)} \tag{5a}$$

$$\langle 2^+|M_B|5^+\rangle = \langle 2^+|V|3^+\rangle \cdot \frac{\langle 3^+|H_B|4^+\rangle}{E(3^+)-E(5^+)} \cdot \frac{\langle 4^+|H_B|5^+\rangle}{E(4^+)-E(5^+)} \cdot \tag{5b}$$

$$\langle 3^+|M_B|5^+\rangle = \langle 3^+|V|4^+\rangle \cdot \frac{\langle 4^+|H_B|5^+\rangle}{E(4^+)-E(5^+)} \tag{5c}$$

The matrix element $\langle 0^+|V|1^+\rangle$ should be approximately equal to that of the mirror transition from the 0^+ ground state of Si^{26} to the 1^+ state of Al^{26}, which has an experimentally measured log ft of 3.5. The matrix element $\langle 2^+|V|3^+\rangle$ cannot be taken directly from another transition like $\langle 0^+|V|1^+\rangle$. Reasonable estimates are obtained by using log ft values of the neighboring decays of the 3^+ ground state of Na^{26} to the same 2^+ state of Mg^{26} with a log ft of 4.7 and of the 3^+ ground state of Al^{28} to the 2^+ state of Si^{28} with a log ft of 4.9.

Even if the magnetic matrix elements in (5a) are smaller than the energy denominators by factors of 2 or 3, and the four electromagnetic factors reduce the transition matrix element by two orders of magnitude, the result will still enhance the overall process by several orders of magnitude over the observed decay which has a log ft of 14.2. The magnetic matrix elements in

(5b) can be smaller than the energy denominators by considerable factors and still enhance the overall process by several orders of magnitude. The transition (5c) is not considered further because of its very low energy.

Rough quantitative estimates of the expression (5) are obtainable from the shell model description of the states in Al^{26} as a neutron and a proton in the $d_{5/2}$ shell coupled to spins 1, 2, 3, 4 and 5. We can use the experimental magnetic moments of the $5/2^+$ ground states of the nuclei Mg^{25} and Al^{25}; namely -0.9 n.m. and +3.6 n.m. respectively, as values for the effective magnetic moments of the $5/2^+$ neutron and the $5/2^+$ proton configurations in Al^{26}. We therefore need assume only that the states of spins 1-5 in Al^{26} are described by different couplings of the neutron configuration of Mg^{25} and the proton configuration of Al^{25}, without assuming a particular model like a single-particle description for either.

The electromagnetic transition operator H_B and its relevant matrix elements can then be written

$$H_B = (g_p j_{pz} + g_n j_{nz})B_z = (g_p+g_n)J_z B_z/2 + (g_p-g_n)(j_{pz}-j_{nz})B_z/2 \qquad (6a)$$

$$\langle J|H_B|J+1\rangle = \langle J|j_{pz}-j_{nz}|J+1\rangle \ \{0.9(e\hbar/2Mc)B_z\} \qquad (6b)$$

where g_p and g_n denote the gyromagnetic ratios of the proton and neutron configurations, j_{pa} and j_{nz} the z-components of the angular momenta of these configurations, B_z the magnetic field strength, chosen to be in the z-direction and J_z the z-component of the total angular momentum. The values of g_p and g_n were taken from the experimental moments,

$$(g_p-g_n) = (2/5)(3.6 + 0.9)(e\hbar/2Mc) = 0.9(e\hbar/Mc) \qquad (6c)$$

Using closure and eq. (6b) to evaluate matrix products gives

$$\langle 1^+|H_B|2^+\rangle\langle 2^+|H_B|3^+\rangle\langle 3^+|H_B|4^+\rangle\langle 4^+|H_B|5^+\rangle$$
$$= \langle 1^+|(j_{pz}-j_{nz})^4|5^+\rangle\{0.9(e\hbar/2Mc)B_z\}^4 \tag{7a}$$

$$\langle 3^+|H_B|4^+\rangle\langle 4^+|H_B|5^+\rangle$$
$$= \langle 3^+|(j_{pz}-j_{nz})^2|5^+\rangle\{0.9(e\hbar/2Mc)B_z\}^2 \tag{7b}$$

Substituting (7) and the experimental energy denominators into eqs. (5) and taking for B_z the field of a monopole with $eg/\hbar c = 1$ then gives

$$\langle 0^+|M_B|5^+\rangle = \langle 0^+|V|1^+\rangle\langle 1^+|(j_{pz}-j_{nz})^4|5^+\rangle\{1/2100)(\hbar c/r)^2\}^4/1.6 \tag{8a}$$
$$\langle 2^+|M_B 5^+\rangle = \langle 2^+|V|3^+\rangle\langle 3^+|(j_{pz}-j_{nz})^2|5^+\rangle\{(1/2100)(\hbar c/r)^2\}^2/0.8 \tag{8b}$$

The angular momentum matrix elements are easily evaluated by standard methods. Assuming equal populations for the 11 J_z states and using the values log ft = 3.5 and 4.9 respectively for the two beta transitions we obtain

$$\log ft(5^+\to 0^+) = 7.4 + 16 \log r \tag{9a}$$
$$\log ft(5^+\to 2^+) = 6.8 + 8 \log r \tag{9b}$$

where r is in fermis.

The $(5^+\to 0^+)$ transition is seen to have log ft values of 7.4 and 12.3 for values of r of 1 and 2 fermis respectively. The $(5^+\to 2^+)$ transition (9b) has log ft values of 6.8, 9.2 and 10.6 for values of r of 1, 2 and 3 fermis respectively. These should be compared with the log ft of 14.2 for the competing observed decay.

These very crude estimates only indicate orders of magnitude. Better calculations can be made with time-dependent magnetic fields to account for the passage of a monopole by a nucleus, and with more complicated nuclear wave functions, but these are probably not worth the effort until more information is available about monopoles.

Another kind of metastable state whose decay might be enhanced by the presence of a monopole is the spontaneous fission of a medium or heavy nucleus which could break up exothermally into two smaller nuclei but is hindered by the Coulomb barrier. The barrier would be reduced by the magnetic interaction of the monopole, which has a longer range than the nuclear force. Consider the change in energy produced by placing a spin zero nucleus in a strong magnetic field and orienting all the nucleon magnetic moments parallel to the field. Since this makes all proton spins parallel and all neutron spins parallel, the spatial wave functions must change to antisymmetric states allowed by the Pauli principle. When the gain in magnetic energy is greater than the loss in nuclear energy due to the change in the spatial wave function, the state with the oriented moments will be the ground state of the monopole-nucleus system.

A crude lower bound for the magnetic field strength required is obtainable from eq. (1). The splitting ΔE is just twice the magnetic energy difference between an oriented neutron-proton state and a spin zero state. The magnetic energy gain for a nucleus with equal numbers of neutrons and protons is then just $\Delta E/4$ per nucleon or 1/2 MeV for a monopole with $eg/\hbar c = 1$ at a distance of 10 fermis and 8 MeV per nucleon at a distance of 2.5 fermis. Since the normal binding energy per nucleon is 8 MeV, the transition certainly occurs before the magnetic energy reaches this value, where it is sufficient to break the nucleus completely into nucleons.

However, changes in the nuclear configuration occur long before such a transition, particularly near the fermi surface. In a j-j coupling shell model the magnetic field splits each j shell, and a level diagram analogous to the Nilsson diagram can be plotted as a function of the magnetic field. The nucleon configuration changes every time an unfilled shell crosses a filled shell. The short range nuclear energy is replaced by long range magnetic energy, thus reducing the resistance of the nucleus to spatial deformations. It becomes easier for the Coulomb repulsion to push protons apart if the binding comes from a long range magnetic force rather than from a short range nuclear force. In the language of the liquid drop model, the magnetically oriented state has a lower surface tension. These effects are highly model dependent and difficult to estimate. However, since barrier penetration factors are exponential, a slight change in the balance of forces between short range nuclear attraction, Coulomb repulsion and long range magnetic attraction can produce large changes in fission lifetimes.

These effects might be useful in monopole detectors. A few monopole-induced decays could not be distinguished from a statistical fluctuation in long-lived natural decays. The induced decay can have a special signature; e.g. the higher β-ray energy of the $5^+ \to 0^+$ decay (5a) of Al^{26} compared to the natural $5^+ \to 2^+$ decay (5b) or the cascade gamma ray following the $5^+ \to 3^+$ decay (5c). An induced decay occurring within a superconducting loop detector [3] in coincidence with a signal of the type observed [3] would give convincing evidence for the passage of a monopole. It might therefore be useful to search for favorable nuclear transitions for such a detector.

These effects might even be of practical interest if the decays of fission products in nuclear waste could be catalyzed by this mechanism, reducing the lifetimes and simplifying the disposal problem.

References

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